Thermal Simulation of Large Rotating Solar Arrays

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The simulation of the thermal impact of large solar arrays on the spacecraft often presents problems. This paper discusses a new method of thermal simulation of large rotating solar arrays. The analytical basis of the method is described and its application to solar simulation testing of the European Orbital Test Satellite (OTS) is presented.

Nomenclature

A = area
ΔA = small area
c_I, c_{II} = functions, defined by Eqs. (25) and (26)
F = view factor
G = integral, defined by Eq. (4)
l = length of cone generatrix
Δl = length of truncated cone generatrix
n = surface normal
r = radius
Δr = radii difference
R = radius
S = connecting vector between two area subelements
T = temperature
x,y,z = coordinates
Δx = solar array thickness
β = angle between solar array plane and radius vector r_3 of a radiator area subelement
γ = half cone angle
ϵ = emissivity
ψ = angle between radius vector r_3 of a radiator area subelement and radius vector r_4 of a truncated cone subelement
$\Delta\lambda$ = reduced length of truncated cone generatrix
$\Delta\rho$ = reduced small radial length

Subscripts and Superscripts

1 = related to solar array area A_1
2 = related to area A_2, see Fig. 2
3 = related to radiator area ring element dA_3
4 = related to truncated cone area element dA_4
a,b = indicates subsections of solar array area
c = related to wall thickness
e = exterior
i = interior
$*$ = indicates area subelements

Introduction

LARGE solar arrays are difficult to simulate thermally because the dimensions of available solar simulation test facilities usually do not allow a full-size spacecraft to be tested in flight configuration. Stub panels are often used to simulate thermally the infrared heat radiated from the full-size solar arrays to the spacecraft body. Obviously, stub panels can account for the total amount of heat only and not for the correct local distribution of heat radiated from the full-size solar arrays to the spacecraft body.

Modern, 3-axis-stabilized, geosynchronous communication satellites, such as for instance OTS (Fig. 1), are particularly sensitive to irradiation from the solar arrays. Thermal control radiators on this type of spacecraft are usually positioned on spacecraft surfaces, which receive a minimum of solar radiation, but, have a rather large view angle to the rotating solar arrays (one revolution per day). Therefore, a correct simulation of heat locally irradiated from the solar arrays to the spacecraft is required.

It is usually sufficient to simulate the average amount of heat per unit time, irradiated by the rotating solar arrays, as the thermal capacities of the satellites are generally high enough to avoid significant temperature cycling. Further, steady state temperatures conditions are preferred in the analytical evaluation of mathematical models. This has led to the use of an electrically heated truncated cone for the thermal simulation of rotating solar arrays. A truncated cone with its axis in the solar array axis of rotation and with its imaginary apex on the spacecraft radiator would agree in one essential point with the local distribution of heat irradiated by a rotating solar array. Assuming that the truncated cone shell and the solar array are infinitely thin, it is obvious that no heat is irradiated by either configuration (indicated in Figs. 2 and 3) to the central point where the axes of cone or solar array, respectively, penetrate the spacecraft radiator. Following this idea, it was presumed that it should be possible to adapt the local heat distribution generated by the truncated cone to that generated by the rotating solar array by proper choice of the cone angle, the generatrix length and the mean radius.

Analysis

The radiative heat exchange between a radiator area subelement dA_3^* and the solar array area A_1 (see Fig. 2) can be determined by applying view factor derivations available in standard literature.[1] The following definitions are established. The solar array area A_1 consists of its partial areas A_{1a} and A_{1b}, whereas the rectangular area A_2 between solar array and radiator area is divided into the partial areas A_{2a} and A_{2b}. The coordinates of these areas and of dA_3^* are z_1, z_2, y_1, r_3 and β, as depicted in Fig. 2. The view factor from dA_3^* to $A_{1a}+A_{2a}$ is found to be

$$F_{dA_3^*-(A_{1a}+A_{2a})} = \frac{1}{2\pi}\left[\arctan\frac{(y_1/r_3)-\cos\beta}{\sin\beta} - \frac{\sin\beta}{[[(z_1/r_3)]^2+\sin^2\beta]]^{1/2}} \times \arctan\frac{(y_1/r_3)-\cos\beta}{[[(z_1/r_3)]^2+\sin^2\beta]]^{1/2}}\right] \tag{1}$$

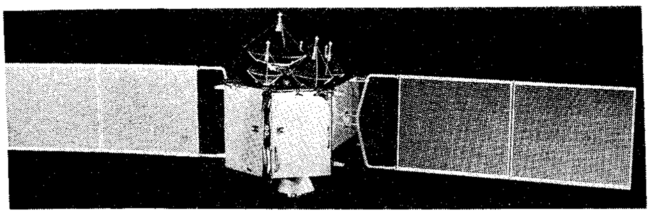

Fig. 1 Model of the OTS satellite.

Received November 5, 1974; revision received December 16, 1974.
Index categories: Radiation and Radiative Heat Transfer; Thermal Modeling and Experimental Thermal Simulation.

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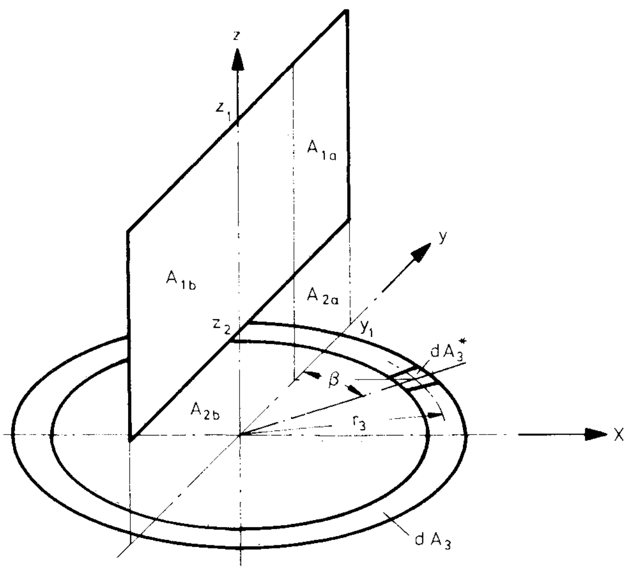

Fig. 2 Geometry of radiator ring element dA_3 and solar array A_1.

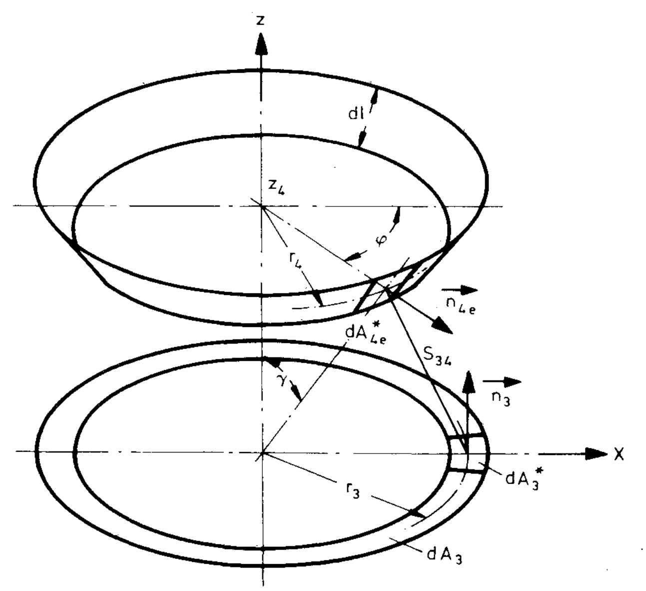

Fig. 3 Geometry of radiator ring element dA_3 and truncated cone element dA_4.

The view factors $F_{dA_3^*-(A_{1b}+A_{2b})}$, $F_{dA_3^*-A_{2a}}$, and $F_{dA_3^*-A_{2b}}$ can be determined accordingly. Finally, the view factor $F_{dA_3^*-A_1}$ from the radiator subelement to the solar array can be derived from

$$F_{dA_3^*-A_1} = F_{dA_3^*-(A_{1a}+A_{2a})} + F_{dA_3^*-(A_{1b}+A_{2b})} - F_{dA_3^*-A_{2a}} - F_{dA_3^*-A_{2b}} \tag{2}$$

and subsequently

$$F_{dA_3^*-A_1} = (1/2\pi)\ [G(z_2,\beta) - G(z_1,\beta)] \tag{3}$$

with

$$G(z_j,\beta) = \sin\beta\left[\left[\frac{z_j}{r_3}\right]^2 + \sin^2\beta\right]^{-1/2} \times \arctan\frac{2(y_1/r_3)\,[[z_j/r_3]^2+\sin^2\beta]^{1/2}}{[z_j/r_3]^2+1-[y_1/r_3]^2} \tag{4}$$

where $j=1$ and $j=2$, respectively, is obtained. The view factor from the radiator ring element dA_3 to the solar array is then found by integration; i.e.,

$$F_{dA_3-A_1} = \frac{1}{dA_3}\int_{dA_3} F_{dA_3^*-A_1}\, dA_3^* \tag{5}$$

or, employing Eqs. (3) and (4),

$$F_{dA_3-A_1} = \frac{1}{\pi^2}\left[\int_0^{\pi/2} G(z_2,\beta)d\beta - \int_0^{\pi/2} G(z_1,\beta)d\beta\right] \tag{6}$$

As the solar array panel has a certain thickness Δx, the lower edge area A_{1c} of the solar array with its normal in $-z$ direction shall be taken into account, too. The view factor from dA_{1c}^*, an area subelement of A_{1c} located at (o,y,z_2), to the circular area A_3 with the radius r_3 can be found e.g., in Ref. 1, to be

$$F_{dA_{1c}^*-A_3} = \tfrac{1}{2}\left[1 - \frac{y^2+z_2^2-r_3^2}{[(y^2+z_2^2-r_3^2)^2+4z_2r_3]^{1/2}}\right] \tag{7}$$

From Eq. (7) the view factor from dA_{1c}^* to the circular ring element dA_3 can be derived by differentiation, i.e.

$$dF_{dA_{1c}^*-dA_3} = (\delta F_{dA_{1c}-dA_3}/\delta r_3)dr_3 \tag{8}$$

which yields

$$dF_{dA_{1c}^*-dA_3} = 2\,z_2^2 r_3 \times \frac{y^2+z_2^2+r_3^2}{[(y^2+z_2^2-r_3^2)^2+4\,z_2^2r_3^2]^{3/2}}\,dr_3 \tag{9}$$

Applying the reciprocity law for view factors,

$$dF_{dA_3-dA_{1c}^*} = \frac{z_2^2\cdot dx}{\pi} \times \frac{y^2+z_2^2+r_3^2}{[(y^2+z_2^2-r_3^2)^2+4\,z_2^2r_3^2]^{3/2}}\,dy \tag{10}$$

is obtained.

Admitting a small panel thickness Δx and a corresponding finite area ΔA_{1c} in place of the differentials dx and dA_{1c}, respectively, the view factor from ring element dA_3 to the lower edge area of the solar array is finally found by integrating Eq. (10),

$$F_{dA_3-A_{1c}} = \frac{2\,z_2^2\Delta x}{\pi} \times \int_0^{y_1} \frac{y^2+z_2^2+r_3^2}{[(y^2+z_2^2-r_3^2)^2+4\,z_2^2r_3^2]^{3/2}}\,dy \tag{11}$$

The sum of $F_{dA_3-A_1}$, [Eq. (6)], and $F_{dA_3-A_{1c}}$, [Eq. (11)] represents the total view factor for irradiation from a solar array, identified by y_1, z_1, z_2 and Δx, to a radiator area ring element and can be numerically determined in dependence of the radius r_3 of the ring element.

The view factor $F_{dA_3-dA_4}$ between the outer surface of a truncated cone element dA_{4e}, defined by radius r_4, half cone angle γ and the length of generatrix dl_4 (see Fig. 3), and a radiator area ring element dA_3 can be derived by vector algebra. Vectors r_3, r_4 and z_4 describe the locations of subelements dA_3^* and dA_{4e}^* in the coordinate system indicated in Fig. 3. The connecting vector from dA_{4e}^* to dA_3^* is

$$S_{34} = \begin{bmatrix} r_4\cos\psi - r_3 \\ r_4\sin\psi \\ z_4 \end{bmatrix} \tag{12}$$

and the surface normals are

$$n_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{13}$$

and

$$n_{4e} = \begin{bmatrix} \cos\psi \cos\gamma \\ \sin\psi \cos\gamma \\ -\sin\gamma \end{bmatrix} \tag{14}$$

The view factor between dA_3^* and $dA_{4\hat{e}}$ can be calculated by

$$dF_{dA_3^* - dA_{4\hat{e}}} = \frac{n \cdot S_{34} \cdot n_{4e} \cdot S_{43}}{\pi S_{34}{}^4} dA_{4\hat{e}} \tag{15}$$

and, finally, with

$$dA_{4\hat{e}} = r_4 d\psi\, dl \tag{16}$$

$$c_I = ½[(r_4/r_3)\ (1+\cot^2\gamma) + (r_3/r_4)] \tag{17}$$

and

$$c_{II} = \frac{\cot^2\gamma}{4\pi r_3} \cdot \sin\gamma \tag{18}$$

$$dF_{dA_3^* - dA_{4\hat{e}}} = c_{II} dl \left[\frac{1}{\cos\psi - c_I} + \frac{c_I}{(\cos\psi - c_I)^2} \right] d\psi \tag{19}$$

Integration of Eq. (19) leads to view factor $dF_{dA_3^* - dA_{4e}}$, which for symmetry reasons is identical with view factor $dF_{dA_3 - dA_{4e}}$, that is

$$dF_{dA_3 - dA_{4e}} = 2 \int_{\psi=0}^{\psi=\pi/2} dF_{dA_3^* - dA_{4\hat{e}}} \tag{20}$$

and, explicitly,

$$dF_{dA_3 - dA_{4e}} = \frac{2c_{II} dl}{c_I{}^2 - 1} \left[1 + 2[c_I{}^2 - 1]^{-½} \cdot \arctan\left[\frac{c_I + 1}{c_I - 1} \right]^{½} \right] \tag{21}$$

View factor $dF_{dA_3 - dA_{4i}}$ from the radiator area ring element dA_3 to the inner surface of the truncated cone element dA_{4i} can be derived analogously by taking into account that for the surface normals of the truncated cone subelement

$$n_{4i} = -n_{4e} \tag{22}$$

holds. Thus,

$$dF_{dA_3 - dA_{4i}} = -2\, c_{II} dl \times \int_{\psi=\pi/2}^{\psi=\pi} \left[\frac{1}{\cos\psi - c_I} + \frac{c_I}{(\cos\psi - c_I)^2} \right] d\psi \tag{23}$$

and, finally,

$$dF_{dA_3 - dA_{4i}} = \frac{2c_{II} dl}{c_I{}^2 - 1} \left[1 + 2[c_I{}^2 - 1]^{-½} \cdot \left[\arctan\left[\frac{c_I + 1}{c_I - 1} \right]^{½} - \frac{\pi}{2} \right] \right] \tag{24}$$

is obtained.

Equations (21) and (24) are also applicable to a truncated cone with the finite generatrix length Δl, if $\Delta l \ll S_{34}$ holds.

The view factor from a subelement $dA_{4\hat{c}}$ of the lower edge element area $dA_{4\hat{c}}$ of the truncated cone, the normal of which is pointing in $-z$ direction, to the ring element area dA_3 can be taken from Eq. (9) and is expressed by

$$dF_{dA_{4\hat{c}} - dA_3} = 2\, z_{4c}{}^2 \cdot r_3 \times \frac{r_{4c}{}^2 + z_{4c}{}^2 + r_3{}^2}{[(r_{4c}{}^2 + z_{4c}{}^2 - r_3{}^2)^2 + 4\, z_{4c}{}^2 r_3{}^2]^{3/2}} dr_3 \tag{25}$$

where r_{4c} and z_{4c} are found from

$$r_{4c} = r_4 - \Delta l/2\ \sin\gamma \tag{26}$$

and

$$z_{4c} = r_{4c}/\tan\gamma \tag{27}$$

For symmetry reasons,

$$dF_{dA_{4c} - dA_3} = dF_{dA_{4\hat{c}}^* - dA_3} \tag{28}$$

$$\times \frac{r_{4c}{}^2 + z_{4c}{}^2 + r_3{}^2}{[(r_{4c}{}^2 + z_{4c}{}^2 - r_3{}^2)^2 + 4\, z_{4c}{}^2 r_3{}^2]^{3/2}}\ dr_{4c} \tag{29}$$

is found. Providing the difference between outer and inner radius Δr_{4c} of a finite ring area ΔA_{4c} is small, in particular $\Delta r_{4c} \ll S_{34}$, Eq. (29) is applicable to these geometries, too. By solving the equation

$$F_{dA_3 - A_I}(r_3) + F_{dA_3 - \Delta A_{Ic}}(r_3) = F_{dA_3 - \Delta A_{4e}}(r_3) + F_{dA_3 - \Delta A_{4i}}(r_3) + F_{dA_3 - \Delta A_{4c}}(r_3) \tag{30}$$

for γ, r_4, Δl and Δr_{4c} at four different radii r_3, an agreement between the total view factor of the solar array and of the truncated cone at these four radii r_3 can be accomplished.

An even better approximation of the heat irradiated to the radiator area in an interesting range $0 \le r_3 \le R_3$ can be achieved, if γ, r_4, Δl, and Δr_{4c} are determined by employing a computer program, taking into account that the total amounts of heat irradiated from the solar array or from the truncated cone, respectively, to the radiator area, are identical, that is

$$\int_{r_3=0}^{r_3=R_3} [F_{dA_3 - A_I} + F_{dA_3 - \Delta A_{Ic}}] dA_3 = \int_{r_3=0}^{r_3=R_3} [F_{dA_3 - \Delta A_{4e}} + F_{dA_3 - \Delta A_{4i}} + F_{dA_3 - \Delta A_{4c}}] dA_3 \tag{31}$$

and that the local adjustment of irradiated heat is optimized, that is

$$\int_{r_3=0}^{r_3=R_3} [F_{dA_3 - A_I} + F_{dA_3 - \Delta A_{Ic}} - F_{dA_3 - \Delta A_{4e}} - F_{dA_3 - \Delta A_{4i}} - F_{dA_3 - \Delta A_{4c}}]^2 \cdot r_3{}^2 dr_3 = \text{Min!} \tag{32}$$

Numerical solution of Eq. (32) also leads to the relatively best approximation achievable, if additional constraints, as for instance test chamber dimensional constraints, have to be regarded.

Assuming "gray" surface—which is tolerable, as only radiation within a narrow infrared range is under consideration—the dimensions of the truncated cone can be reduced from Δl to $\Delta\lambda$ and from Δr_{4c} to $\Delta\rho_{4c}$, respectively, the heat input to the radiator ring element remaining unchanged, if

$$\Delta\lambda/\Delta l = \Delta\rho_{4c}/\Delta r_{4c} = \epsilon_I T_I{}^4/\epsilon_4 T_4{}^4 \tag{33}$$

is observed, where ϵ stands for emissivity and T for temperature.

Thermal Simulation of the OTS Rotating Solar Array

For the OTS spacecraft two solar arrays, the dimensions of which are: $z_i = 3903$ mm; $z_2 = 941$ mm; $y_I = 637$ mm; and

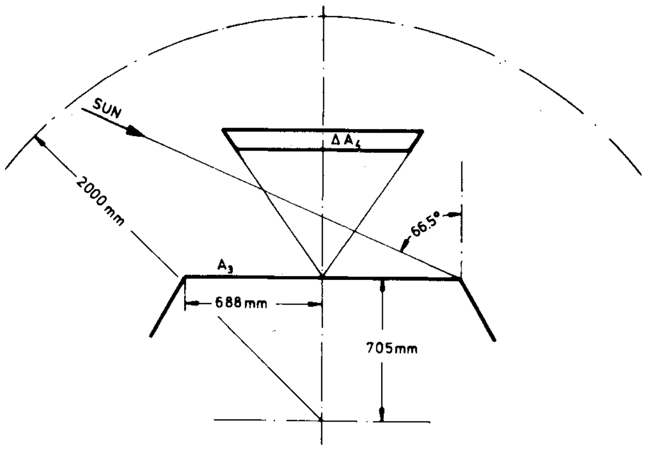

Fig. 4 Truncated cone ΔA_4 adapted to spatial constraints of test facility.

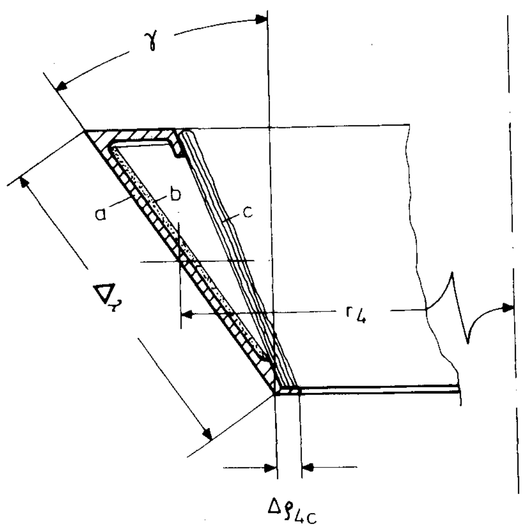

Fig. 5 Design of heated truncated cone: a = aluminum frame; b = heater mat, bonded to frame; and c = multilayer insulation.

$\Delta x = 18.6$ mm, have to be thermally simulated. The spatial restrictions of the SIMLES solar simulation test facility of CNES, Toulouse, are presented in Fig. 4.

The truncated cone thermally simulating one of the solar arrays has to remain within a spherical envelope—radius 2000 mm—in order to avoid interference with the gimbal system of the test chamber. Furthermore, shadowing of the radiator A_3 by the truncated cone A_4 has to be avoided, when the simulated solar light is impinging on the radiator area A_3 at an incident angle of 66.5°. In order to obtain a light and yet rigid truncated cone for OTS, it was decided to employ radiation from the outer surface A_{4e} only and to insulate the inner surface A_{4i} (see Fig. 5). The aluminum truncated cone frame is equipped with a heater mat on the inside and coated black on the outside. The frame is supported by four thin, thermally decoupled struts with highly reflective coating, which are mounted to the spacecraft at the solar array latching points. The ratio $\epsilon_1 T_1^4/\epsilon_4 T_4^4$ is chosen to be 0.6, which for a solar array temperature of $T_1 = 60°$C leads to a

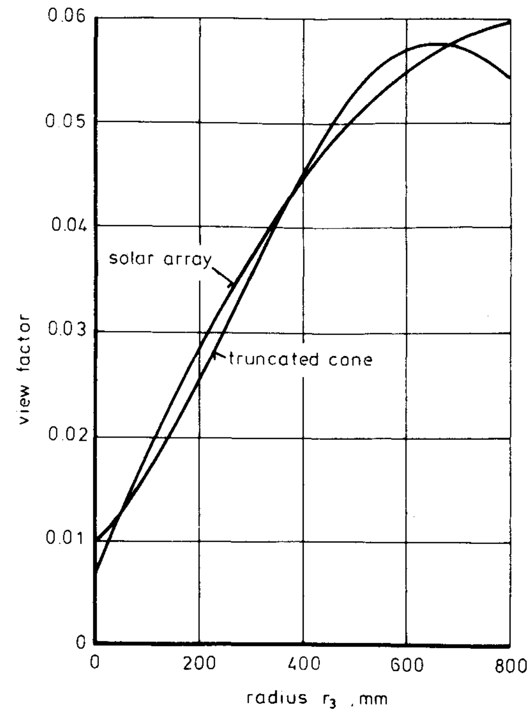

Fig. 6 Comparison of view factors from radiator ring element to solar array and truncated cone, respectively.

truncated cone temperature of approximately $T_4 = 95°$C, ($\epsilon_1 = 0.8$; $\epsilon_4 = 0.9$).

With regard to the aforementioned restrictions and design data, the numerical evaluation of Eqs. (31) and (32) resulted in $\Delta\lambda = 114 \pm 0.05$ mm; $\Delta\rho_{4c} = 5.54 \pm 0.1$ mm; $r_4 = 464.78 \pm 1$ mm; $z_4 = 682.62 \pm 1$ mm; and $\gamma = 34.25 \pm 0.2°$.

The view factors from a radiator ring element area dA_3 to a truncated cone of nonreduced dimensions and to the OTS solar array, respectively, are plotted vs r_3 in Fig. 6. Up to $R_3 = 700$ mm, the difference between the two view factor functions is nowhere higher than 5.5% of the actual maximum value (see Fig. 6). The total view factor from A_3 to the truncated cone deviates by only +0.06% from the total view factor from A_3 to the solar array.

Summary

A method of thermally simulating large rotating solar arrays is developed. The view factor functions for the geometries radiator/solar array and radiator/truncated cone, respectively, are derived. For the OTS spacecraft these functions are evaluated and compared, and the specific truncated cone design features are determined and described. The heated truncated cone is found to be adaptable to spatial restrictions of the test facility and yet to be able to sufficiently accurately simulate the local and the total heat irradiation from the OTS solar array to the OTS thermal control radiator.

References

[1] Sparrow, E. M. and Cess, R. D., *Radiation Heat Transfer*, Brooks/Cole, Belmont, Calif. 1970, pp. 300-305.